Mathematics Chapter - 01

* Students are Advised to Solve the Questions of Exercises in the same Sequence 'or' as Directed by the **Faculty Members.** *

Conceptual Notes for NTSE/KVPY/Olympiad/Boards

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~ Key Concept ~

Number System

- **Natural Numbers :**
  The simplest numbers are 1, 2, 3, 4....... the numbers being used in counting. These are called natural numbers.
- **Whole numbers :**
  The natural numbers along with the zero form the set of whole numbers i.e. numbers 0, 1, 2, 3, 4 are whole numbers. W = {0, 1, 2, 3, 4....}
- **Integers :**
  The natural numbers, their negatives and zero make up the integers. Z = {....–4, –3, –2, –1, 0, 1, 2, 3, 4,....}
  The set of integers contains positive numbers, negative numbers and zero.
- **Rational Number :**
  (i) A rational number is a number which can be put in the form p/q, where p & q are both integers & $q \neq 0$.
  (ii) A rational number is either a terminating or non-terminating and recurring (repeating) decimal.
  (iii) A rational number may be positive, negative or zero.
- **Complex numbers :**
  Complex numbers are imaginary numbers of the form a + ib, where a and b are real numbers and $i = \sqrt{-1}$, which is an imaginary number.
- **Factors :**
  A number is a factor of another, if the former exactly divides the latter without leaving a remainder (remainder is zero) 3 and 5 are factors of 12 and 25 respectively.
- **Multiples :**
  A multiple is a number which is exactly divisible by another, 36 is a multiple of 2, 3, 4, 9 and 12.
- **Even Numbers :**
  Integers which are multiples of 2 are even number (i.e.) 2,4, 6, 8.............. are even numbers.
- **Odd numbers :**
  Integers which are not multiples of 2 are odd numbers.
- **Prime and composite Numbers :**
  All natural number which cannot be divided by any number other than 1 and itself is called a prime number. By convention, 1 is not a prime number. 2, 3, 5, 7, 11, 13, 17 .... are prime numbers. Numbers which are not prime are called composite numbers.
- **The Absolute Value (or modulus) of a real Number :**
  If a is a real number, modulus a is written as $|a|$; $|a|$ is always positive or zero. It means positive value of 'a' whether a is positive or negative
  $|3| = 3$ and $|0| = 0$,
  Hence $|a| = a$; if a = 0 or a > 0 (i.e.) $a \geq 0$
  $|-3| = 3 = -(-3)$. Hence $|a| = -a$ when $a < 0$
  Hence, $|a| = a$, if $a > 0$; $|a| = -a$, if $a < 0$
- **Irrational number :**
  (i) A number is irrational if and only if its decimal representation is non-terminating and non-repeating. e.g. $\sqrt{2}$, $\sqrt{3}$, π................ etc.
  (ii) Rational number and irrational number taken together form the set of real numbers.

(iii) If a and b are two real numbers, then either (i) a > b or (ii) a = b or (iii) a < b

(iv) Negative of an irrational number is an irrational number.

(v) The sum of a rational number with an irrational number is always irrational.

(vi) The product of a non-zero rational number with an irrational number is always an irrational number.

(vii) The sum of two irrational numbers is not always an irrational number.

(viii) The product of two irrational numbers is not always an irrational number.

In division for all rationales of the form p/q (q ≠ 0), p & q are integers, two things can happen either the remainder becomes zero or never becomes zero.

Type (1) Eg : $\frac{7}{8}$ = 0.875

$$
\begin{array}{r}
8\,\overline{)\,70\,}\,(0.875 \\
64 \\ \hline
60 \\
56 \\ \hline
40 \\
40 \\ \hline
\times
\end{array}
$$

This decimal expansion 0.875 is called **terminating**.

∴ If remainder is zero then decimal expansion ends (terminates) after finite number of steps. These decimal expansion of such numbers terminating.

Type (2) Eg : $\frac{1}{3}$ = 0.333……… = $0.\bar{3}$

$$
\begin{array}{r}
3\,\overline{)\,10\,}\,(0.33\ldots\ldots \\
9 \\ \hline
10 \\
9 \\ \hline
1\ldots\ldots
\end{array}
$$

In both examples remainder is never becomes zero so the decimal expansion is never ends after some or infinite steps of division. These type of decimal expansions are called **non terminating.**

In above examples, after I[st] step & 6 steps of division (respectively) we get remainder equal to dividend so decimal expansion is repeating (recurring).

So these are called **non terminating recurring decimal expansions**.

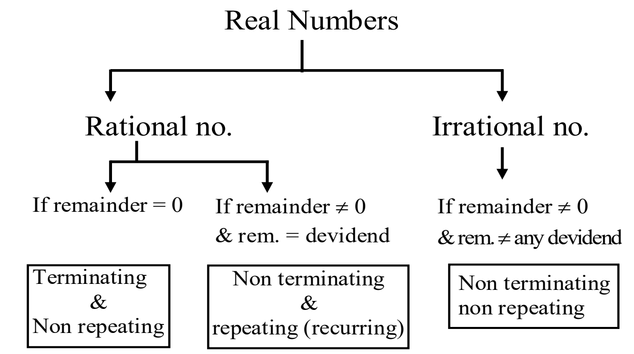
Both the above types (1 & 2) are rational numbers.

Types (3) Eg : The decimal expansion 0.327172398……is not ends anywhere, also there is no arrangement of digits (not repeating) so these are called **non terminating not recurring**.

These numbers are called **irrational numbers**.

Eg. :

0.1279312793	rational	terminating
0.1279312793…. or $0.\overline{12793}$	rational	non terminating & recurring
0.32777	rational	terminating
$0.32\bar{7}$ or0.32777…….	rational	non terminating & recurring
0.5361279	rational	terminating
0.3712854043….	irrational	non terminating non recurring
0.10100100010000	rational	terminating
0.10100100010000….	irrational	non terminating non recurring.



Eg : $\frac{18}{5} = 3.6$ Eg : $\frac{1}{3} = 0.33\ldots.. = 0.\bar{3}$ Eg : 0.671234…..

❖ Examples ❖

Ex.1 Insert a rational and an irrational number between 2 and 3.

Sol. If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

Also, if a,b are rational numbers, then $\frac{a+b}{2}$ is a rational number between them.

∴ A rational no. between 2 and 3 is $\frac{2+3}{2}$ = 2.5

An irrational no. between 2 and 3 is $\sqrt{2\times 3} = \sqrt{6}$

Ex.2 Find two irrational numbers between 2 and 2.5.

Sol. If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

∴Irrational no. between 2 and 2.5 is $\sqrt{2\times 2.5} = \sqrt{5}$

Similarly, irrational no. between 2 & $\sqrt{5}$ is $\sqrt{2\times\sqrt{5}}$

So, required numbers are $\sqrt{5}$ and $\sqrt{2\times\sqrt{5}}$.

Ex.3 Find two irrational no. lying between $\sqrt{2}$ & $\sqrt{3}$.

Sol. We know that, if a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.

∴ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is

$\sqrt{\sqrt{2}\times\sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$

Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2}\times 6^{1/4}}$ = $2^{1/4} \times 6^{1/8}$.

Hence required irrational number are $6^{1/4}$ and $2^{1/4} \times 6^{1/8}$.

Ex.4 Find two irrational numbers between 0.12 and 0.13.

Sol. Let a = 0.12 and b = 0.13. Clearly, a and b are rational numbers such that a < b.

We observe that the number a and b have a 1 in the first place of decimal. But in the second place of decimal a has a 2 and b has 3. So, we consider the numbers

c = 0.1201001000100001 ……

and, d = 0.12101001000100001…….

Clearly, c and d are irrational numbers such that a < c < d < b.

Theorem : Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Proof : Let the prime factorisation of a be as follows :

a = $p_1p_2\ldots..p_n$, where $p_1,p_2,\ldots..p_n$ are primes, not necessarily distinct. Therefore,

$a^2 = (p_1p_2.....p_n)\ (p_1p_2 ..... p_n) = p_1^2 p_2^2 ..... p_n^2$.

Now, we are given that p divides a^2. Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2. However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we realise that the only prime factors of a^2 are $p_1, p_2,..., p_n$.

So p is one of $p_1, p_2,......, p_n$.

Now, since $a = p_1 p_2 ...... p_n$, p divides a.

We are now ready to give a proof that $\sqrt{2}$ is irrational.

The proof is based on a technique called 'proof by contradiction'.

Ex.5 Prove that $7-\sqrt{3}$ is irrational

Sol. **Method I :** Let $7-\sqrt{3}$ is rational number

$\therefore\ 7-\sqrt{3} = \frac{p}{q}$ (p, q are integers, $q \neq 0$)

$\therefore\ 7-\frac{p}{q} = \sqrt{3} \quad \Rightarrow \quad \sqrt{3} = \frac{7q-p}{q}$

Here p, q are integers $\therefore\ \frac{7q-p}{q}$ is also integer

$\therefore$ LHS = $\sqrt{3}$ is also integer but this is contradiction that $\sqrt{3}$ is irrational so our assumption is wrong that $(7-\sqrt{3})$ is rational

$\therefore\ 7-\sqrt{3}$ is irrational proved.

Method II : Let $7-\sqrt{3}$ is rational

we know sum or difference of two rational is also rational $\therefore\ 7-(7-\sqrt{3}) = \sqrt{3}$ = rational

but this is contradiction that $\sqrt{3}$ is irrational

$\therefore\ (7-\sqrt{3})$ is irrational proved.

➤ Euclid's Division Lemma

'or' Euclid's division Algorithm

For any two positive integers **a** and **b**, there exist unique integers q and r satisfying a = bq + r, where $0 \leq r < b$.

For Example

(i) Consider number 23 and 5, then : 23 = 5 × 4 + 3

Comparing with a = bq + r; we get:

a = 23, b = 5, q = 4, r = 3

and $0 \leq r < b$ (as $0 \leq 3 < 5$).

(ii) Consider positive integers 18 and 4. 18 = 4 × 4 + 2

$\Rightarrow$ For 18 (= a) and 4(= b) we have q = 4, r = 2 and $0 \leq r < b$.

In the relation a = bq + r, where $0 \leq r < b$ is nothing but a statement of the long division of number a by number b in which q is the quotient obtained and r is the remainder. Thus,

dividend = divisor × quotient + remainder $\Rightarrow$ a = bq + r

◈ H.C.F. (Highest Common Factor)

The H.C.F. of two or more positive integers is the largest positive integer that divides each given positive number completely.

i.e., if positive integer **d** divides two positive integers **a** and **b** then the H.C.F. of **a** and **b** is **d**.

For Example

(i) 14 is the largest positive integer that divides 28 and 70 completely; therefore H.C.F. of 28 and 70 is 14.

(ii) H.C.F. of 75, 125 and 200 is 25 as 25 divides each of 75, 125 and 200 completely and so on.

◈ Using Euclid's Division Lemma For Finding H.C.F.

Consider positive integers 418 and 33.

Step-1 : Taking bigger number (418) as **a** and smaller number (33) as **b**

express the numbers as a = bq + r

$\Rightarrow$ 418 = 33 × 12 + 22

Step-2 : Now taking the divisor 33 and remainder 22; apply the Euclid's division algorithm to get:

33 = 22 × 1 + 11 [Expressing as a = bq + r]

Step-3 : Again with new divisor 22 and new remainder 11; apply the Euclid's division algorithm to get:

22 = 11 × 2 + 0

Step-4 : Since, the remainder = 0 so we cannot proceed further.

Step-5 : The last divisor is 11 and we say H.C.F. of 418 and 33 = 11

Verification :

(i) Using factor method:

$\therefore$ Factors of 418 = 1, 2, 11, 19, 22, 38, 209 & 418

and, Factor of 33 = 1, 3, 11 and 33.

Common factors = 1 and 11

$\Rightarrow$ Highest common factor = 11 i.e., H.C.F. = 11

(ii) Using prime factor method:

Prime factors of 418 = 2, 11 and 19.

Prime factors of 33 = 3 and 11.

$\therefore$ **H.C.F.** = Product of all common prime factors = 11.

For any two positive integers a and b which can be expressed as a = bq + r, where $0 \leq r < b$, the, H.C.F. of (a, b) = H.C.F. of (q, r) and so on. For number 418 and 33

418 = 33 × 12 + 22 33 = 22 × 1 + 11

and 22 = 11 × 2 + 0

$\Rightarrow$ H.C.F. of (418, 33) = H.C.F. of (33, 22)

= H.C.F. of (22, 11) = 11.

❖ Examples ❖

Ex.6 Using Euclid's division algorithm, find the H.C.F. of

(i) 135 and 225 (ii) 196 and 38220

Sol.(i) Starting with the larger number i.e., 225, we get:

225 = 135 × 1 + 90

Now taking divisor 135 and remainder 90, we get

135 = 90 × 1 + 45

Further taking divisor 90 and remainder 45, we get

90 = 45 × 2 + 0

$\therefore$ **Required H.C.F. = 45**

(ii) Starting with larger number 38220, we get:

38220 = 196 × 195 + 0

Since, the remainder is 0

$\Rightarrow$ **H.C.F. = 196**

Ex.7 Show that every positive integer is of the form 2q and that every positive odd integer is of the from 2q + 1, where q is some integer.

Sol. According to Euclid's division lemma, if a and b are two positive integers such that a is greater than b; then these two integers can be expressed as

a = bq + r; where $0 \leq r < b$

Now consider

b = 2; then a = bq + r will reduce to

a = 2q + r; where $0 \leq r < 2$,

i.e., r = 0 or r = 1

If r = 0, a = 2q + r $\Rightarrow$ a = 2q

i.e., a is even

and, if r = 1, a = 2q + r $\Rightarrow$ a = 2q + 1

i.e., a is add;

as if the integer is not even; it will be odd.

Since, a is taken to be any positive integer so it is applicable to the every positive integer that when it can be expressed as $a = 2q$

$\therefore$ **a** is even and when it can expressed as $a = 2q + 1$; a is odd.

Hence the required result.

Ex.8 Show that any positive integer which is of the form $6q + 1$ or $6q + 3$ or $6q + 5$ is odd, where q is some integer.

Sol. If **a** and **b** are two positive integers such that **a** is greater than **b**; then according to Euclid's division algorithm; we have $a = bq + r$;

where, q and r are positive integers and $0 \le r < b$.

Let b = 6, then

$a = bq + r \Rightarrow$ **a = 6q + r**; where $0 \le r < 6$.

When $r = 0 \Rightarrow a = 6q + 0 = 6q$;

which is even integer

When $r = 1 \Rightarrow a = 6q + 1$

which is odd integer

When $r = 2 \Rightarrow a = 6q + 2$ **which is even.**

When $r = 3 \Rightarrow a = 6q + 3$ **which is odd.**

When $r = 4 \Rightarrow a = 6q + 4$ **which is even.**

When $r = 5 \Rightarrow a = 6q + 5$ **which is odd.**

This verifies that when r = 1 or 3 or 5; the integer obtained is $6q + 1$ or $6q + 3$ or $6q + 5$ and each of these integers is a positive odd number.

Hence the required result.

Ex.9 Use Euclid's Division Algorithm to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol. Let **a** and **b** are two positive integers such that **a** is greater than **b**; then: $a = bq + r$;

where q and r are also positive integers and $0 \le r < b$

Taking b = 3, we get: $a = 3q + r$; where $0 \le r < 3$

$\Rightarrow$ The value of positive integer **a** will be $3q + 0$, $3q + 1$ or $3q + 2$ i.e., $3q$, $3q + 1$ or $3q + 2$.

Now we have to show that the squares of positive integers $3q$, $3q + 1$ and $3q + 2$ can be expressed as 3m, or 3m + 1 for some integer m.

$\therefore$ **Square of 3q** = $(3q)^2$

$= 9q^2 = 3(3q^2) = 3m$; 3 where m is some integer.

Square of 3q + 1 = $(3q + 1)^2$ $= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1 = 3m + 1$ for some integer m.

Square of 3q + 2 = $(3q + 2)^2$ $= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1 =$ **3m + 1** for some integer m.

$\therefore$ The square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Hence the required result.

➤ The Fundamental Theorem of Arithmetic

Statement : Every composite number can be decomposed as a product prime numbers in a unique way, except for the order in which the prime numbers occur.

For example :

(i) $30 = 2 \times 3 \times 5, 30 = 3 \times 2 \times 5, 30 = 2 \times 5 \times 3$ and so on.

(ii) $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$ or $432 = 3^3 \times 2^4$

(iii) $12600 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$

In general, a composite number is expressed as the product of its prime factors written in ascending order of their values.

Ex. $6615 = 3 \times 3 \times 3 \times 5 \times 7 \times 7$ $= 3^3 \times 5 \times 7^2$

❖ Examples ❖

Ex.10 Consider the number 6^n, where n is a natural number. Check whether there is any value of $n \in N$ for which 6^n is divisible by 7.

Sol. Since, $6 = 2 \times 3$; $6^n = 2^n \times 3^n$

$\Rightarrow$ The prime factorisation of given number 6^n

$\Rightarrow$ **6^n is not divisible by 7.**

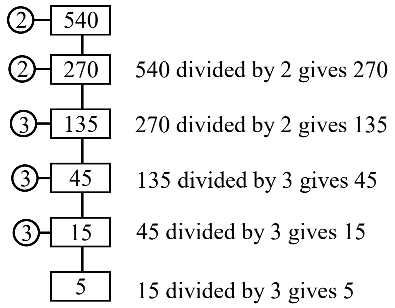
➤ Using the Factor Tree

❖ Examples ❖

Ex.11 Find the prime factors of :

(i) 540 (ii) 21252

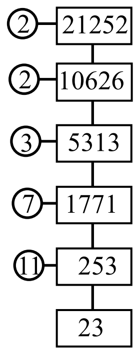
(i)



5 is a prime number and so cannot be further divided by any prime number

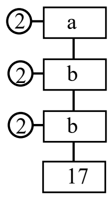
$\therefore$ $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^3 \times 5$

(ii)



$\therefore 21252 = 2 \times 2 \times 3 \times 7 \times 11 \times 23 = \mathbf{2^2 \times 3 \times 11 \times 7 \times 23}$

Ex.12 Find the missing numbers a, b and c in the following factorisation:



Can you find the number on top without finding the other?

Sol. $c = 17 \times 2 = 34$ $b = c \times 2 = 34 \times 2 = 68$

and $a = b \times 2 = 68 \times 2 = 136$

i.e., a = 136, b = 68 and c = 34.

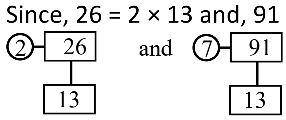
➤ Using the Fundamental Theorem of Arithmetic to find H.C.F. & L.C.M.

❖ Examples ❖

Ex.13 Find the L.C.M. and H.C.F. of the following pairs of integers by applying the Fundamental theorem of Arithmetic method i.e., using the prime factorisation method.

(i) 26 and 91 (ii) 1296 and 2520

Sol. (i) Since, $26 = 2 \times 13$ and, $91 = 7 \times 13$



$\therefore$ **L.C.M.** = Product of each prime factor with highest powers. $= 2 \times 13 \times 7 = 182$. i.e., **L.C.M.** (26, 91) = 182.

H.C.F. = Product of common prime factors with lowest powers. = 13. i.e., **H.C.F** (26, 91) = 13.

(ii) Since, $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^4 \times 3^4$

and, $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2^3 \times 3^2 \times 5 \times 7$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
	3

2	2520
2	1260
2	630
3	315
3	105
5	35
	7

$\therefore$ **L.C.M.** = Product of each prime factor with highest powers

$= 2^4 \times 3^4 \times 5 \times 7 = \mathbf{45,360}$

i.e., **L.C.M.** (1296, 2520) = 45,360

H.C.F. = Product of common prime factors with lowest powers. $= 2^3 \times 3^2 = 8 \times 9 = 72$

i.e., H.C.F. (1296, 2520) = **72.**

> ➤ For any two positive integers :
> Their L.C.M. × their H.C.F.
> = Product of the number
>
> $\Rightarrow$ (i) L.C.M. $= \dfrac{\text{Product of the numbers}}{\text{H.C.F.}}$
>
> (ii) H.C.F. $= \dfrac{\text{Product of the numbers}}{\text{L.C.M.}}$
>
> (iii) One number $= \dfrac{\text{L.C.M.} \times \text{H.C.F.}}{\text{Other number}}$

Ex.14 Given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100)

Sol. L.C.M. (150, 100) = 300

$\Rightarrow$ L.C.M. of 150 and 100 = 300

Since, the product of number 150 and 100 = 150 × 100

we know : H.C.F. (150, 100) $= \dfrac{\text{Product of 150 and 100}}{\text{L.C.M.(150,100)}}$

$= \dfrac{150 \times 100}{300} = 50.$

Ex.15 The H.C.F. and L.C.M. of two numbers are 12 and 240 respectively. If one of these numbers is 48; find the other numbers.

Sol. Since, the product of two numbers

= Their H.C.F. × Their L.C.M.

$\Rightarrow$ One no. × other no. = H.C.F. × L.C.M.

$\Rightarrow$ Other no. $= \dfrac{12 \times 240}{48} = 60.$

➤ High Order Thinking Problems

Q.3 Classify following numbers as rational or irrational :

(i) 22/7 (ii) 3.1416 (iii) π (iv) $3.\overline{142857}$

(v) 5.636363...... (vi) 2.040040004......

(vii) $\sqrt{21}$ (viii) $\sqrt[3]{3}$

Q.4 Prove that each of the following numbers is irrational

(i) $\sqrt{6}$ (ii) $(2-\sqrt{3})$ (iii) $(3+\sqrt{2})$

(iv) $(5+3\sqrt{2})$ (v) $\dfrac{3}{\sqrt{5}}$ (vi) $3\sqrt{7}$

Q.6 Without actual division, show that each of the following rational numbers is a non-terminating repeating decimal :

(i) $\dfrac{11}{(2^3 \times 3)}$ (ii) $\dfrac{73}{(2^3 \times 3^3 \times 5)}$ (iii) $\dfrac{9}{35}$

(iv) $\dfrac{32}{147}$ (v) $\dfrac{64}{455}$ (vi) $\dfrac{77}{210}$

Q.7 Without actual divison, show that each of the following rational numbers is a terminating decimal. Express each in decimal form :

(i) $\dfrac{23}{(2^3 \times 5^2)}$ (ii) $\dfrac{24}{125}$ (iii) $\dfrac{17}{320}$

(iv) $\dfrac{171}{800}$ (v) $\dfrac{15}{1600}$ (vi) $\dfrac{19}{3125}$

Q.8 Express each of the following as a fraction in simplest form :

(i) $0.\bar{8}$ (ii) $2.\bar{4}$ (iii) $0.\overline{24}$

(iv) $0.1\bar{2}$ (v) $2.2\bar{4}$ (vi) $0.3\overline{65}$

Q.9 Decide whether the given number is rational or not :

(i) 53.123456789 (ii) $31.\overline{123456789}$

(iii) 0.12012001200012...

Give reason to support your answer.

Q.10 What do you mean by Euclid's division algorithm.

Q.11 A number when divided by 61 gives 27 as quotient and 32 as remainder. Find the number.

Q.12 By what number should 1365 be divided to get 31 as quotient and 32 as remainder ?

Q.13 Using Euclid's algorithm, find the HCF of

(i) 405 and 2520 (ii) 504 and 1188

(iii) 960 and 1575

Q.14 Using prime factorisation, find the HCF and LCM of

(i) 144, 198 (ii) 396, 1080

Q.15 Using prime factorisation, find the HCF and LCM of

(i) 24, 36, 40 (ii) 30, 72, 432 (iii) 21, 28, 36, 45

Q.16 The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

Q.17 The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, find the other.

Q.18 Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank ?

Q.20 Find the maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils.

❖ Answers ❖

3. (i) Rational (ii) Rational (iii) Irrational
(iv) Rational (v) Rational (vi) Irrational
(ix) Irrational (x) Irrational

7. (i) 0.115 (ii) 0.192 (iii) 0.053125
(iv) 0.21375 (v) 0.009375 (vi) 0.00608

8. (i) $\dfrac{8}{9}$ (ii) $\dfrac{22}{9}$ (iii) $\dfrac{8}{33}$
(iv) $\dfrac{11}{90}$ (v) $\dfrac{101}{45}$ (vi) $\dfrac{181}{495}$

9. (i) Rational, since it is a terminating decimal
(ii) Rational, since it is a repeating decimal
(iii) Not rational, since it is a non-terminating & non-repeating decimal

11. 1679 **12.** 43 **13.** (i) 45 (ii) 36 (iii) 15

14. (i) HCF = 18, LCM = 1584 (ii) HCF = 36, LCM = 11880
(iii) HCF = 128, LCM = 14976

15. (i) HCF = 4, LCM = 360 (ii) HCF = 6, LCM = 2160
(iii) HCF = 1, LCM = 1260

16. 207 **17.** 308 **18.** 7 m **19.** 35 cm **20.** 91

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☎आपका परिश्रम + हमारा मार्गदर्शन = निश्चित सफलता☎

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*** **With Best Wishes** ***